

## 5.1 Alternative Assignments

Day #1 &amp; Day #2

An even longer strategy to verify  $\frac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$ , but one that works, is to replace each of the two occurrences of  $\csc^2 x$  on the left side by  $\frac{1}{\sin^2 x}$ . This may be the approach that you first consider, particularly if you become accustomed to rewriting the more complicated side in terms of sines and cosines. The selection of an appropriate fundamental identity to solve the puzzle most efficiently is learned through lots of practice.

Day #1

1, 2, 6, 7, 11, 13, 15, 21, 23, 25,  
27, 29, 31, 35, 39, 41, 43

Day #2

3, 4, 8, 10, 14, 16, 20, 24, 26, 30,  
32, 36, 40, 44, 47, 53, 59\*, 60\*, 61, 62, 63, 64\*

Guidelines for Verifying Trigonometric Identities

The more identities you prove, the more confident and efficient you will become. Although practice is the only way to learn how to verify identities, there are some guidelines developed throughout the section that should help you get started.

\*Bonus

Your choice!

Math XL

or these assignments on paper.

- Work with each side of the equation independently of the other side. Start with the more complicated side and transform it in a step-by-step fashion until it looks exactly like the other side.
  - Analyze the identity and look for opportunities to apply the fundamental identities.
  - Try using one or more of the following techniques:
    1. Rewrite the more complicated side in terms of sines and cosines.
    2. Factor out the greatest common factor.
    3. Separate a single-term quotient into two terms:
- $$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad \text{and} \quad \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$
- 4. Combine fractional expressions using the least common denominator.
  - 5. Multiply the numerator and the denominator by a binomial factor that appears on the other side of the identity.
  - Don't be afraid to stop and start over again if you are not getting anywhere. Creative puzzle solvers know that strategies leading to dead ends often provide good problem-solving ideas.

## CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

1. To verify an identity, start with the more \_\_\_\_\_ side and transform it in a step-by-step fashion until it is identical to the \_\_\_\_\_ side.
2. It is sometimes helpful to verify an identity by rewriting one of the sides in terms of \_\_\_\_\_ and \_\_\_\_\_, and then simplifying the result.
3. True or false: To verify the identity

$$\frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x},$$

we should begin by multiplying both sides by  $\sin x(1 + \cos x)$ , the least common denominator. \_\_\_\_\_

4.  $\frac{1}{\csc x - 1} - \frac{1}{\csc x + 1}$  can be simplified using \_\_\_\_\_ as the least common denominator.
5. You can use your graphing calculator to provide evidence of an identity. Graph the left side and the right side separately, and see if the two graphs are \_\_\_\_\_.

## EXERCISE SET 5.1

## Practice Exercises

In Exercises 1–60, verify each identity.

1.  $\sin x \sec x = \tan x$
2.  $\cos x \csc x = \cot x$
3.  $\tan(-x) \cos x = -\sin x$
4.  $\cot(-x) \sin x = -\cos x$
5.  $\tan x \csc x \cos x = 1$
6.  $\cot x \sec x \sin x = 1$
7.  $\sec x - \sec x \sin^2 x = \cos x$
8.  $\csc x - \csc x \cos^2 x = \sin x$
9.  $\cos^2 x - \sin^2 x = 1 - 2 \sin^2 x$
10.  $\cos^2 x - \sin^2 x = 2 \cos^2 x - 1$
11.  $\csc \theta - \sin \theta = \cot \theta \cos \theta$
12.  $\tan \theta + \cot \theta = \sec \theta \csc \theta$

$$(13) \frac{\tan \theta \cot \theta}{\csc \theta} = \sin \theta$$

$$(15) \sin^2 \theta (1 + \cot^2 \theta) = 1$$

$$(17) \sin t \tan t = \frac{1 - \cos^2 t}{\cos t}$$

$$(19) \frac{\csc^2 t}{\cot t} = \csc t \sec t$$

$$(21) \frac{\tan^2 t}{\sec t} = \sec t - \cos t$$

$$(14) \frac{\cos \theta \sec \theta}{\cot \theta} = \tan \theta$$

$$(16) \cos^2 \theta (1 + \tan^2 \theta) = 1$$

$$(18) \cos t \cot t = \frac{1 - \sin^2 t}{\sin t}$$

$$(20) \frac{\sec^2 t}{\tan t} = \sec t \csc t$$

$$(22) \frac{\cot^2 t}{\csc t} = \csc t - \sin t$$

13.  $\frac{1 - \cos \theta}{\sin \theta} = \csc \theta - \cot \theta$     24.  $\frac{1 - \sin \theta}{\cos \theta} = \sec \theta - \tan \theta$

25.  $\frac{\sin t + \cos t}{\csc t + \sec t} = 1$

26.  $\frac{\sin t + \cos t}{\tan t + \cot t} = \sin t + \cos t$

27.  $\tan t + \frac{\cos t}{1 + \sin t} = \sec t$

28.  $\cot t + \frac{\sin t}{1 + \cos t} = \csc t$

29.  $1 - \frac{\sin^2 x}{1 + \cos x} = \cos x$

30.  $1 - \frac{\cos^2 x}{1 + \sin x} = \sin x$

31.  $\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = 2 \sec x$

32.  $\frac{\sin x}{\cos x + 1} + \frac{\cos x - 1}{\sin x} = 0$

33.  $\sec^2 x \csc^2 x = \sec^2 x + \csc^2 x$

34.  $\csc^2 x \sec x = \sec x + \csc x \cot x$

35.  $\frac{\sec x - \csc x}{\sec x + \csc x} = \frac{\tan x - 1}{\tan x + 1}$     36.  $\frac{\csc x - \sec x}{\csc x + \sec x} = \frac{\cot x - 1}{\cot x + 1}$

37.  $\frac{\sin^2 x - \cos^2 x}{\sin x + \cos x} = \sin x - \cos x$

38.  $\frac{\tan^2 x - \cot^2 x}{\tan x + \cot x} = \tan x - \cot x$

39.  $\tan^2 2x + \sin^2 2x + \cos^2 2x = \sec^2 2x$

40.  $\cot^2 2x + \cos^2 2x + \sin^2 2x = \csc^2 2x$

41.  $\frac{\tan 2\theta + \cot 2\theta}{\csc 2\theta} = \sec 2\theta$     42.  $\frac{\tan 2\theta + \cot 2\theta}{\sec 2\theta} = \csc 2\theta$

43.  $\frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$

44.  $\frac{\cot x + \cot y}{1 - \cot x \cot y} = \frac{\cos x \sin y + \sin x \cos y}{\sin x \sin y - \cos x \cos y}$

45.  $(\sec x - \tan x)^2 = \frac{1 - \sin x}{1 + \sin x}$

46.  $(\csc x - \cot x)^2 = \frac{1 - \cos x}{1 + \cos x}$

47.  $\frac{\sec t + 1}{\tan t} = \frac{\tan t}{\sec t - 1}$     48.  $\frac{\csc t - 1}{\cot t} = \frac{\cot t}{\csc t + 1}$

49.  $\frac{1 + \cos t}{1 - \cos t} = (\csc t + \cot t)^2$

50.  $\frac{\cos^2 t + 4 \cos t + 4}{\cos t + 2} = \frac{2 \sec t + 1}{\sec t}$

51.  $\cos^4 t - \sin^4 t = 1 - 2 \sin^2 t$

52.  $\sin^4 t - \cos^4 t = 1 - 2 \cos^2 t$

53.  $\frac{\sin \theta - \cos \theta}{\sin \theta} + \frac{\cos \theta - \sin \theta}{\cos \theta} = 2 - \sec \theta \csc \theta$

54.  $\frac{\sin \theta}{1 - \cot \theta} - \frac{\cos \theta}{\tan \theta - 1} = \sin \theta + \cos \theta$

55.  $(\tan^2 \theta + 1)(\cos^2 \theta + 1) = \tan^2 \theta + 2$

56.  $(\cot^2 \theta + 1)(\sin^2 \theta + 1) = \cot^2 \theta + 2$

57.  $(\cos \theta - \sin \theta)^2 + (\cos \theta + \sin \theta)^2 = 2$

58.  $(3 \cos \theta - 4 \sin \theta)^2 + (4 \cos \theta + 3 \sin \theta)^2 = 25$

59.  $\frac{\cos^2 x - \sin^2 x}{1 - \tan^2 x} = \cos^2 x$     BONUS

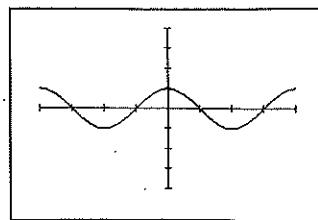
60.  $\frac{\sin x + \cos x}{\sin x} - \frac{\cos x - \sin x}{\cos x} = \sec x \csc x$

BONUS.

### Practice Plus

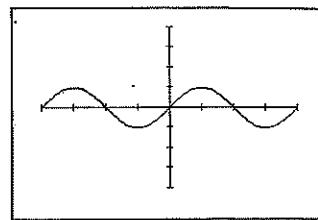
In Exercises 61–66, half of an identity and the graph of this half are given. Use the graph to make a conjecture as to what the right side of the identity should be. Then prove your conjecture.

61.  $\frac{(\sec x + \tan x)(\sec x - \tan x)}{\sec x} = ?$



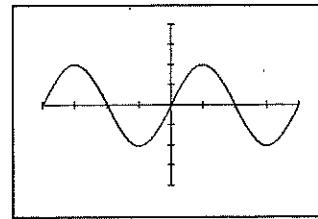
$[-2\pi, 2\pi, \frac{\pi}{2}]$  by  $[-4, 4, 1]$

62.  $\frac{\sec^2 x \csc x}{\sec^2 x + \csc^2 x} = ?$



$[-2\pi, 2\pi, \frac{\pi}{2}]$  by  $[-4, 4, 1]$

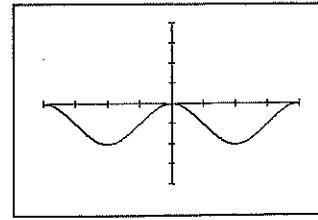
63.  $\frac{\cos x + \cot x \sin x}{\cot x} = ?$



$[-2\pi, 2\pi, \frac{\pi}{2}]$  by  $[-4, 4, 1]$

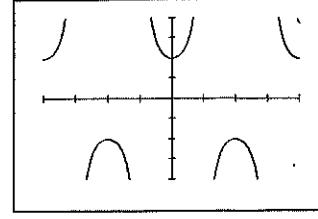
64.  $\frac{\cos x \tan x - \tan x + 2 \cos x - 2}{\tan x + 2} = ?$

BONUS



$[-2\pi, 2\pi, \frac{\pi}{2}]$  by  $[-4, 4, 1]$

65.  $\frac{1}{\sec x + \tan x} + \frac{1}{\sec x - \tan x} = ?$



$[-2\pi, 2\pi, \frac{\pi}{2}]$  by  $[-4, 4, 1]$

**DISCOVERY**

Derive the sum and difference formulas for tangents by working Exercises 55 and 56 in Exercise Set 5.2.

5.2 Alternate Assigns

5.2 Day 1: 1, 5, 9, 11, 13, 15, 25,  
27, 29, 33, 37, 41,  
49, 51, 57, 59

5.2 Day 2: 2, 3, 6, 7, 8, 10, 12,  
14, 16, 20, 22, 26,  
28, 32, 48, 50,  
52, 56, 58, 60,  
63

**Sum and Difference Formulas for Tangents**

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

The tangent of the sum of two angles equals the tangent of the first angle plus the tangent of the second angle divided by 1 minus their product.

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

The tangent of the difference of two angles equals the tangent of the first angle minus the tangent of the second angle divided by 1 plus their product.

**EXAMPLE 7 Verifying an Identity**

Verify the identity:  $\tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - 1}{\tan x + 1}$ .

**SOLUTION**

We work with the left side.

$$\begin{aligned} \tan\left(x - \frac{\pi}{4}\right) &= \frac{\tan x - \tan \frac{\pi}{4}}{1 + \tan x \tan \frac{\pi}{4}} & \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \\ &= \frac{\tan x - 1}{1 + \tan x \cdot 1} & \tan \frac{\pi}{4} = 1 \\ &= \frac{\tan x - 1}{\tan x + 1} \end{aligned}$$

 Check Point 7 Verify the identity:  $\tan(x + \pi) = \tan x$ .

**CONCEPT AND VOCABULARY CHECK**

Fill in each blank so that the resulting statement is true.

1.  $\cos(x + y) =$  \_\_\_\_\_
2.  $\cos(x - y) =$  \_\_\_\_\_
3.  $\sin(C + D) =$  \_\_\_\_\_
4.  $\sin(C - D) =$  \_\_\_\_\_
5.  $\tan(\theta + \phi) =$  \_\_\_\_\_

6.  $\tan(\theta - \phi) =$  \_\_\_\_\_

7. True or false: The cosine of the sum of two angles equals the sum of the cosines of those angles. \_\_\_\_\_
8. True or false:  $\tan 75^\circ = \tan 30^\circ + \tan 45^\circ$  \_\_\_\_\_

**EXERCISE SET 5.2****Practice Exercises**

Use the formula for the cosine of the difference of two angles to solve Exercises 1–12.

In Exercises 1–4, find the exact value of each expression.

1.  $\cos(45^\circ - 30^\circ)$

2.  $\cos(120^\circ - 45^\circ)$

3.  $\cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$

4.  $\cos\left(\frac{2\pi}{3} - \frac{\pi}{6}\right)$

In Exercises 5–8, each expression is the right side of the formula for  $\cos(\alpha - \beta)$  with particular values for  $\alpha$  and  $\beta$ .

- Identify  $\alpha$  and  $\beta$  in each expression.
- Write the expression as the cosine of an angle.
- Find the exact value of the expression.

5.  $\cos 50^\circ \cos 20^\circ + \sin 50^\circ \sin 20^\circ$

6.  $\cos 50^\circ \cos 5^\circ + \sin 50^\circ \sin 5^\circ$

## 5.2 Alternate Assignments, cont'd

Section 5.2 Sum and Difference Formulas 669

7.  $\cos \frac{5\pi}{12} \cos \frac{\pi}{12} + \sin \frac{5\pi}{12} \sin \frac{\pi}{12}$

8.  $\cos \frac{5\pi}{18} \cos \frac{\pi}{9} + \sin \frac{5\pi}{18} \sin \frac{\pi}{9}$

In Exercises 9–12, verify each identity.

9.  $\frac{\cos(\alpha - \beta)}{\cos \alpha \sin \beta} = \tan \alpha + \cot \beta$

10.  $\frac{\cos(\alpha - \beta)}{\sin \alpha \sin \beta} = \cot \alpha \cot \beta + 1$

11.  $\cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x + \sin x)$

12.  $\cos\left(x - \frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}(\cos x + \sin x)$

Use one or more of the six sum and difference identities to solve Exercises 13–54.

In Exercises 13–24, find the exact value of each expression.

13.  $\sin(45^\circ - 30^\circ)$

14.  $\sin(60^\circ - 45^\circ)$

15.  $\sin 105^\circ$

16.  $\sin 75^\circ$

17.  $\cos(135^\circ + 30^\circ)$

18.  $\cos(240^\circ + 45^\circ)$

19.  $\cos 75^\circ$

20.  $\cos 105^\circ$

21.  $\tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

22.  $\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

23.  $\tan\left(\frac{4\pi}{3} - \frac{\pi}{4}\right)$

24.  $\tan\left(\frac{5\pi}{3} - \frac{\pi}{4}\right)$

In Exercises 25–32, write each expression as the sine, cosine, or tangent of an angle. Then find the exact value of the expression.

25.  $\sin 25^\circ \cos 5^\circ + \cos 25^\circ \sin 5^\circ$

26.  $\sin 40^\circ \cos 20^\circ + \cos 40^\circ \sin 20^\circ$

27.  $\frac{\tan 10^\circ + \tan 35^\circ}{1 - \tan 10^\circ \tan 35^\circ}$

28.  $\frac{\tan 50^\circ - \tan 20^\circ}{1 + \tan 50^\circ \tan 20^\circ}$

29.  $\sin \frac{5\pi}{12} \cos \frac{\pi}{4} - \cos \frac{5\pi}{12} \sin \frac{\pi}{4}$

30.  $\sin \frac{7\pi}{12} \cos \frac{\pi}{12} - \cos \frac{7\pi}{12} \sin \frac{\pi}{12}$

31.  $\frac{\tan \frac{\pi}{5} - \tan \frac{\pi}{30}}{1 + \tan \frac{\pi}{5} \tan \frac{\pi}{30}}$

32.  $\frac{\tan \frac{\pi}{5} + \tan \frac{4\pi}{5}}{1 - \tan \frac{\pi}{5} \tan \frac{4\pi}{5}}$

In Exercises 33–54, verify each identity.

33.  $\sin\left(x + \frac{\pi}{2}\right) = \cos x$

34.  $\sin\left(x + \frac{3\pi}{2}\right) = -\cos x$

35.  $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

36.  $\cos(\pi - x) = -\cos x$

37.  $\tan(2\pi - x) = -\tan x$

38.  $\tan(\pi - x) = -\tan x$

39.  $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cos \beta$

40.  $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cos \beta$

41.  $\frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta} = \tan \alpha - \tan \beta$

42.  $\frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta} = \tan \alpha + \tan \beta$

43.  $\tan\left(\theta + \frac{\pi}{4}\right) = \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$

44.  $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}$

45.  $\cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \beta - \sin^2 \alpha$

46.  $\sin(\alpha + \beta) \sin(\alpha - \beta) = \cos^2 \beta - \cos^2 \alpha$

47.  $\frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \frac{\tan \alpha + \tan \beta}{\tan \alpha - \tan \beta}$

48.  $\frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} = \frac{1 - \tan \alpha \tan \beta}{1 + \tan \alpha \tan \beta}$

49.  $\frac{\cos(x+h) - \cos x}{h} = \cos x \frac{\cos h - 1}{h} - \sin x \frac{\sin h}{h}$

50.  $\frac{\sin(x+h) - \sin x}{h} = \cos x \frac{\sin h}{h} + \sin x \frac{\cos h - 1}{h}$

51.  $\sin 2\alpha = 2 \sin \alpha \cos \alpha$

*Hint:* Write  $\sin 2\alpha$  as  $\sin(\alpha + \alpha)$ .

52.  $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$

*Hint:* Write  $\cos 2\alpha$  as  $\cos(\alpha + \alpha)$ .

53.  $\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$

*Hint:* Write  $\tan 2\alpha$  as  $\tan(\alpha + \alpha)$ .

54.  $\tan\left(\frac{\pi}{4} + \alpha\right) - \tan\left(\frac{\pi}{4} - \alpha\right) = 2 \tan 2\alpha$

*Hint:* Use the result in Exercise 53.

55. Derive the identity for  $\tan(\alpha + \beta)$  using

$$\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)}$$

After applying the formulas for sums of sines and cosine divide the numerator and denominator by  $\cos \alpha \cos \beta$ .

56. Derive the identity for  $\tan(\alpha - \beta)$  using

$\tan(\alpha - \beta) = \tan[\alpha + (-\beta)]$ .

After applying the formula for the tangent of the sum of angles, use the fact that the tangent is an odd function.

In Exercises 57–64, find the exact value of the following under given conditions:

a.  $\cos(\alpha + \beta)$

b.  $\sin(\alpha + \beta)$

c.  $\tan(\alpha + \beta)$

57.  $\sin \alpha = \frac{3}{5}$ ,  $\alpha$  lies in quadrant I, and  $\sin \beta = \frac{5}{13}$ ,  $\beta$  lies in quadrant II.

58.  $\sin \alpha = \frac{4}{5}$ ,  $\alpha$  lies in quadrant I, and  $\sin \beta = \frac{7}{25}$ ,  $\beta$  lies in quadrant II.

59.  $\tan \alpha = -\frac{3}{4}$ ,  $\alpha$  lies in quadrant II, and  $\cos \beta = \frac{1}{3}$ ,  $\beta$  lies in quadrant I.

60.  $\tan \alpha = -\frac{4}{3}$ ,  $\alpha$  lies in quadrant II, and  $\cos \beta = \frac{2}{3}$ ,  $\beta$  lies in quadrant I.

61.  $\cos \alpha = \frac{8}{17}$ ,  $\alpha$  lies in quadrant IV, and  $\sin \beta = -\frac{1}{2}$ ,  $\beta$  lies in quadrant III.

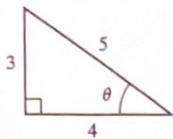
62.  $\cos \alpha = \frac{1}{2}$ ,  $\alpha$  lies in quadrant IV, and  $\sin \beta$  is rest of quadrant III.

63.  $\tan \alpha = \frac{3}{4}$ ,  $\pi < \alpha < \frac{3\pi}{2}$ , and  $\cos \beta = \frac{1}{4}$ ,  $\frac{3\pi}{2} < \beta < 2\pi$ .

64.  $\sin \alpha = \frac{5}{6}$ ,  $\frac{\pi}{2} < \alpha < \pi$ , and  $\tan \beta = \frac{3}{7}$ ,  $\pi < \beta < 2\pi$ .

**EXERCISE SET 5.3****Practice Exercises**

In Exercises 1–6, use the figures to find the exact value of each trigonometric function.



1.  $\sin 2\theta$     2.  $\cos 2\theta$     3.  $\tan 2\theta$   
 4.  $\sin 2\alpha$     5.  $\cos 2\alpha$     6.  $\tan 2\alpha$

In Exercises 7–14, use the given information to find the exact value of each of the following:

- a.  $\sin 2\theta$     b.  $\cos 2\theta$     c.  $\tan 2\theta$ .

7.  $\sin \theta = \frac{15}{17}$ ,  $\theta$  lies in quadrant II.  
 8.  $\sin \theta = \frac{12}{13}$ ,  $\theta$  lies in quadrant II.  
 9.  $\cos \theta = \frac{24}{25}$ ,  $\theta$  lies in quadrant IV.  
 10.  $\cos \theta = \frac{40}{41}$ ,  $\theta$  lies in quadrant IV.  
 11.  $\cot \theta = 2$ ,  $\theta$  lies in quadrant III.  
 12.  $\cot \theta = 3$ ,  $\theta$  lies in quadrant III.  
 13.  $\sin \theta = -\frac{9}{41}$ ,  $\theta$  lies in quadrant III.  
 14.  $\sin \theta = -\frac{2}{3}$ ,  $\theta$  lies in quadrant III.

In Exercises 15–22, write each expression as the sine, cosine, or tangent of a double angle. Then find the exact value of the expression.

15.  $2 \sin 15^\circ \cos 15^\circ$     16.  $2 \sin 22.5^\circ \cos 22.5^\circ$   
 17.  $\cos^2 75^\circ - \sin^2 75^\circ$     18.  $\cos^2 105^\circ - \sin^2 105^\circ$   
 19.  $2 \cos^2 \frac{\pi}{8} - 1$     20.  $1 - 2 \sin^2 \frac{\pi}{12}$   
 21.  $\frac{2 \tan \frac{\pi}{12}}{1 - \tan^2 \frac{\pi}{12}}$     22.  $\frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$

In Exercises 23–34, verify each identity.

23.  $\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$     24.  $\sin 2\theta = \frac{2 \cot \theta}{1 + \cot^2 \theta}$   
 25.  $(\sin \theta + \cos \theta)^2 = 1 + \sin 2\theta$   
 26.  $(\sin \theta - \cos \theta)^2 = 1 - \sin 2\theta$   
 27.  $\sin^2 x + \cos 2x = \cos^2 x$   
 28.  $1 - \tan^2 x = \frac{\cos 2x}{\cos^2 x}$   
 29.  $\cot x = \frac{\sin 2x}{1 - \cos 2x}$   
 30.  $\cot x = \frac{1 + \cos 2x}{\sin 2x}$   
 31.  $\sin 2t - \tan t = \tan t \cos 2t$   
 32.  $\sin 2t - \cot t = -\cot t \cos 2t$   
 33.  $\sin 4t = 4 \sin t \cos^3 t - 4 \sin^3 t \cos t$   
 34.  $\cos 4t = 8 \cos^4 t - 8 \cos^2 t + 1$

**Math 127 Alternate assignments****Power-Reducing**

- 5.3 Day 2  
 #2, 3, 7, 11, 13, 25,  
 29, 39, 43, 50–52, 54, 57, 58

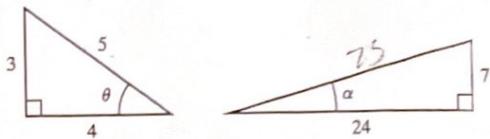
In Exercises 35–38, use the power-reducing formulas to rewrite each expression as an equivalent expression that does not contain powers of trigonometric functions greater than 1.

35.  $6 \sin^4 x$     36.  $10 \cos^4 x$   
 37.  $\sin^2 x \cos^2 x$     38.  $8 \sin^2 x \cos^2 x$

In Exercises 39–46, use a half-angle formula to find the exact value of each expression.

39.  $\sin 15^\circ$     40.  $\cos 22.5^\circ$     41.  $\cos 157.5^\circ$   
 42.  $\sin 105^\circ$     43.  $\tan 75^\circ$     44.  $\tan 112.5^\circ$   
 45.  $\tan \frac{7\pi}{8}$     46.  $\tan \frac{3\pi}{8}$

In Exercises 47–54, use the figures to find the exact value of each trigonometric function.



47.  $\sin \frac{\theta}{2}$     48.  $\cos \frac{\theta}{2}$     49.  $\tan \frac{\theta}{2}$   
 50.  $\sin \frac{\alpha}{2}$     51.  $\cos \frac{\alpha}{2}$     52.  $\tan \frac{\alpha}{2}$   
 53.  $2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$     54.  $2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}$

In Exercises 55–58, use the given information to find the exact value of each of the following:

- a.  $\sin \frac{\alpha}{2}$     b.  $\cos \frac{\alpha}{2}$     c.  $\tan \frac{\alpha}{2}$   
 55.  $\tan \alpha = \frac{4}{3}$ ,  $180^\circ < \alpha < 270^\circ$   
 56.  $\tan \alpha = \frac{8}{15}$ ,  $180^\circ < \alpha < 270^\circ$   
 57.  $\sec \alpha = -\frac{13}{5}$ ,  $\frac{\pi}{2} < \alpha < \pi$   
 58.  $\sec \alpha = -3$ ,  $\frac{\pi}{2} < \alpha < \pi$

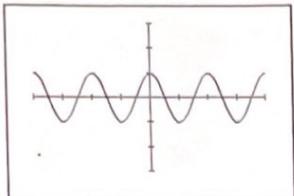
In Exercises 59–68, verify each identity.

59.  $\sin^2 \frac{\theta}{2} = \frac{\sec \theta - 1}{2 \sec \theta}$     60.  $\sin^2 \frac{\theta}{2} = \frac{\csc \theta - \cot \theta}{2 \csc \theta}$   
 61.  $\cos^2 \frac{\theta}{2} = \frac{\sin \theta + \tan \theta}{2 \tan \theta}$     62.  $\cos^2 \frac{\theta}{2} = \frac{\sec \theta + 1}{2 \sec \theta}$   
 63.  $\tan \frac{\alpha}{2} = \frac{\tan \alpha}{\sec \alpha + 1}$   
 64.  $2 \tan \frac{\alpha}{2} = \frac{\sin^2 \alpha + 1 - \cos^2 \alpha}{\sin \alpha(1 + \cos \alpha)}$   
 65.  $\cot \frac{x}{2} = \frac{\sin x}{1 - \cos x}$   
 66.  $\cot \frac{x}{2} = \frac{1 + \cos x}{\sin x}$   
 67.  $\tan \frac{x}{2} + \cot \frac{x}{2} = 2 \csc x$   
 68.  $\tan \frac{x}{2} - \cot \frac{x}{2} = -2 \cot x$

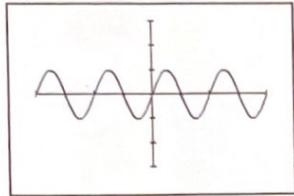
## Practice Plus

In Exercises 69–78, half of an identity and the graph of this half are given. Use the graph to make a conjecture as to what the right side of the identity should be. Then prove your conjecture.

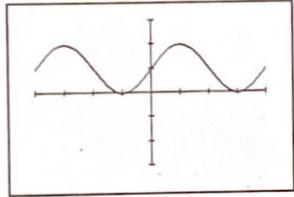
69.  $\frac{\cot x - \tan x}{\cot x + \tan x} = ?$


 $[-2\pi, 2\pi, \frac{\pi}{2}] \text{ by } [-3, 3, 1]$ 

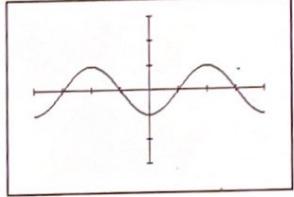
70.  $\frac{2(\tan x - \cot x)}{\tan^2 x - \cot^2 x} = ?$


 $[-2\pi, 2\pi, \frac{\pi}{2}] \text{ by } [-3, 3, 1]$ 

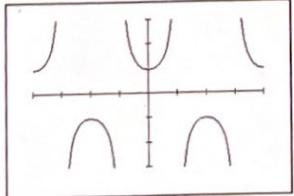
71.  $\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2 = ?$


 $[-2\pi, 2\pi, \frac{\pi}{2}] \text{ by } [-3, 3, 1]$ 

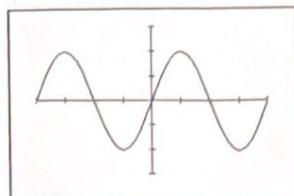
72.  $\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} = ?$


 $[-2\pi, 2\pi, \frac{\pi}{2}] \text{ by } [-3, 3, 1]$ 

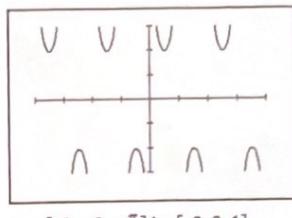
73.  $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = ?$


 $[-2\pi, 2\pi, \frac{\pi}{2}] \text{ by } [-3, 3, 1]$ 

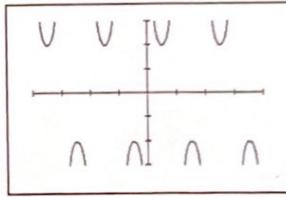
74.  $\sin 2x \sec x = ?$


 $[-2\pi, 2\pi, \frac{\pi}{2}] \text{ by } [-3, 3, 1]$ 

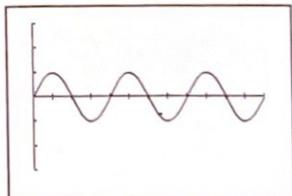
75.  $\frac{\csc^2 x}{\cot x} = ?$


 $[-2\pi, 2\pi, \frac{\pi}{2}] \text{ by } [-3, 3, 1]$ 

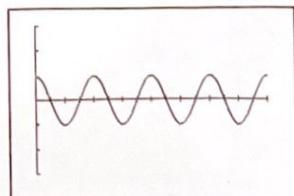
76.  $\tan x + \cot x = ?$


 $[-2\pi, 2\pi, \frac{\pi}{2}] \text{ by } [-3, 3, 1]$ 

77.  $\sin x(4 \cos^2 x - 1) = ?$


 $[0, 2\pi, \frac{\pi}{6}] \text{ by } [-3, 3, 1]$ 

78.  $1 - 8 \sin^2 x \cos^2 x = ?$


 $[0, 2\pi, \frac{\pi}{8}] \text{ by } [-3, 3, 1]$

# Chapter 5 Review

Name \_\_\_\_\_

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Complete the identity.

1)  $\sec x - \frac{1}{\sec x} = ?$

1) \_\_\_\_\_

A)  $\sec x \csc x$

B)  $-2 \tan^2 x$

C)  $1 + \cot x$

D)  $\sin x \tan x$

2)  $\tan(\pi - \theta) = ?$

2) \_\_\_\_\_

A)  $-\tan \theta$

B)  $\cot \theta$

C)  $-\cot \theta$

D)  $\tan \theta$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Use a half-angle formula to find the exact value of the expression.

3)  $\cos \frac{5\pi}{12}$

3) \_\_\_\_\_

Use trigonometric identities to find the exact value.

4) 
$$\frac{\tan 40^\circ + \tan 110^\circ}{1 - \tan 40^\circ \tan 110^\circ}$$

4) \_\_\_\_\_

Solve the equation on the interval  $[0, 2\pi)$ .

5)  $2 \sin^2 x = \sin x$

5) \_\_\_\_\_

Use the given information to find the exact value of the expression.

6)  $\cos \theta = \frac{21}{29}$ ,  $\theta$  lies in quadrant IV      Find  $\sin 2\theta$ .

6) \_\_\_\_\_

7)  $\sin \theta = \frac{24}{25}$ ,  $\theta$  lies in quadrant II      Find  $\tan 2\theta$ .

7) \_\_\_\_\_

Use the given information to find the exact value of the trigonometric function.

8)  $\sec \theta = 4$ ,  $\theta$  lies in quadrant I      Find  $\cos \frac{\theta}{2}$ .

8) \_\_\_\_\_

Verify the identity.

9)  $\csc^2 u - \cos u \sec u = \cot^2 u$

9) \_\_\_\_\_

$$10) \sin(\alpha - \beta) \cos(\alpha + \beta) = \sin \alpha \cos \alpha - \sin \beta \cos \beta$$

$$10) \underline{\hspace{1cm}}$$

$$11) \tan \theta \cdot \csc \theta = \sec \theta$$

$$11) \underline{\hspace{1cm}}$$

$$12) \cos 4\theta = 2 \cos^2(2\theta) - 1 \quad (\text{Start with left-hand side})$$

$$12) \underline{\hspace{1cm}}$$

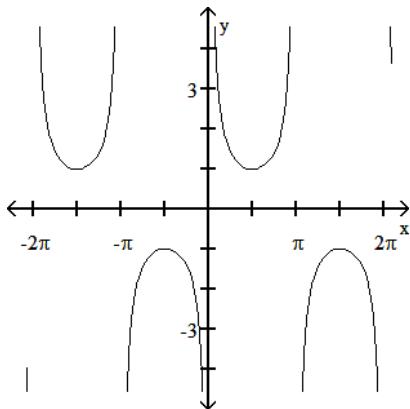
$$13) \sin\left(\frac{3\pi}{2} - \theta\right) = -\cos \theta$$

$$13) \underline{\hspace{1cm}}$$

Use the graph to complete the identity.

$$14) \frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} = ?$$

14) \_\_\_\_\_



Use a half-angle formula to find the exact value of the expression.

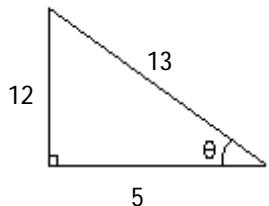
$$15) \tan 105^\circ$$

15) \_\_\_\_\_

Use the figure to find the exact value of the trigonometric function.

$$16) \text{Find } \sin 2\theta.$$

16) \_\_\_\_\_



Find all solutions of the equation.

$$17) \cos x = 0$$

17) \_\_\_\_\_

Solve the equation on the interval  $[0, 2\pi)$ .

$$18) \cot^2 x \cos x = \cot^2 x$$

18) \_\_\_\_\_

Solve the equation on the interval  $[0, 2\pi]$ .

19)  $\sin 3x = 0$

19) \_\_\_\_\_

20)  $\sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = 1$

20) \_\_\_\_\_

21)  $\cos 2x = \frac{\sqrt{2}}{2}$

21) \_\_\_\_\_

22)  $\cos x + 2 \cos x \sin x = 0$

22) \_\_\_\_\_

Solve the problem.

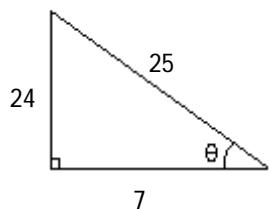
- 23) An airplane flying faster than the speed of sound creates sound waves that form a cone. If  $\alpha$  is the vertex angle of the cone and  $m$  is the Mach number for the speed of the plane, then  $\sin \frac{\alpha}{2} = \frac{1}{m}$  ( $m > 1$ ). Write the formula to calculate the Mach number if  $\alpha = 90^\circ$ .

23) \_\_\_\_\_

Use the figure to find the exact value of the trigonometric function.

24) Find  $\tan 2\theta$ .

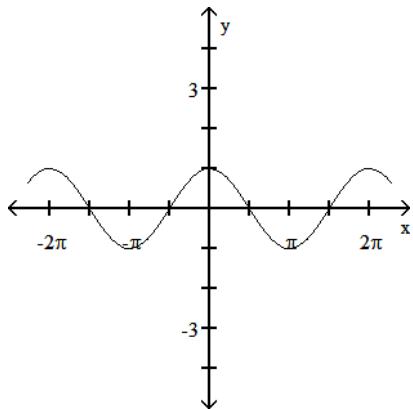
24) \_\_\_\_\_



Use the graph to complete the identity.

25)  $\frac{(\sec x + \tan x)(\sec x - \tan x)}{\sec x} = ?$

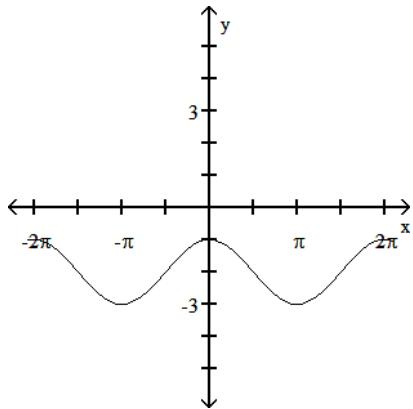
25) \_\_\_\_\_



MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

26)  $\frac{\cos x \tan x - 2\tan x + 5\cos x - 10}{\tan x + 5} = ?$

26) \_\_\_\_\_



A)  $\sin x - 5$

B)  $\cos x - 2$

C)  $\sin x + 5\cos x$

D)  $\cos x + 2$